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ONTARIO

DEPARTMENT OF EDUCATION

MATHEMATICS

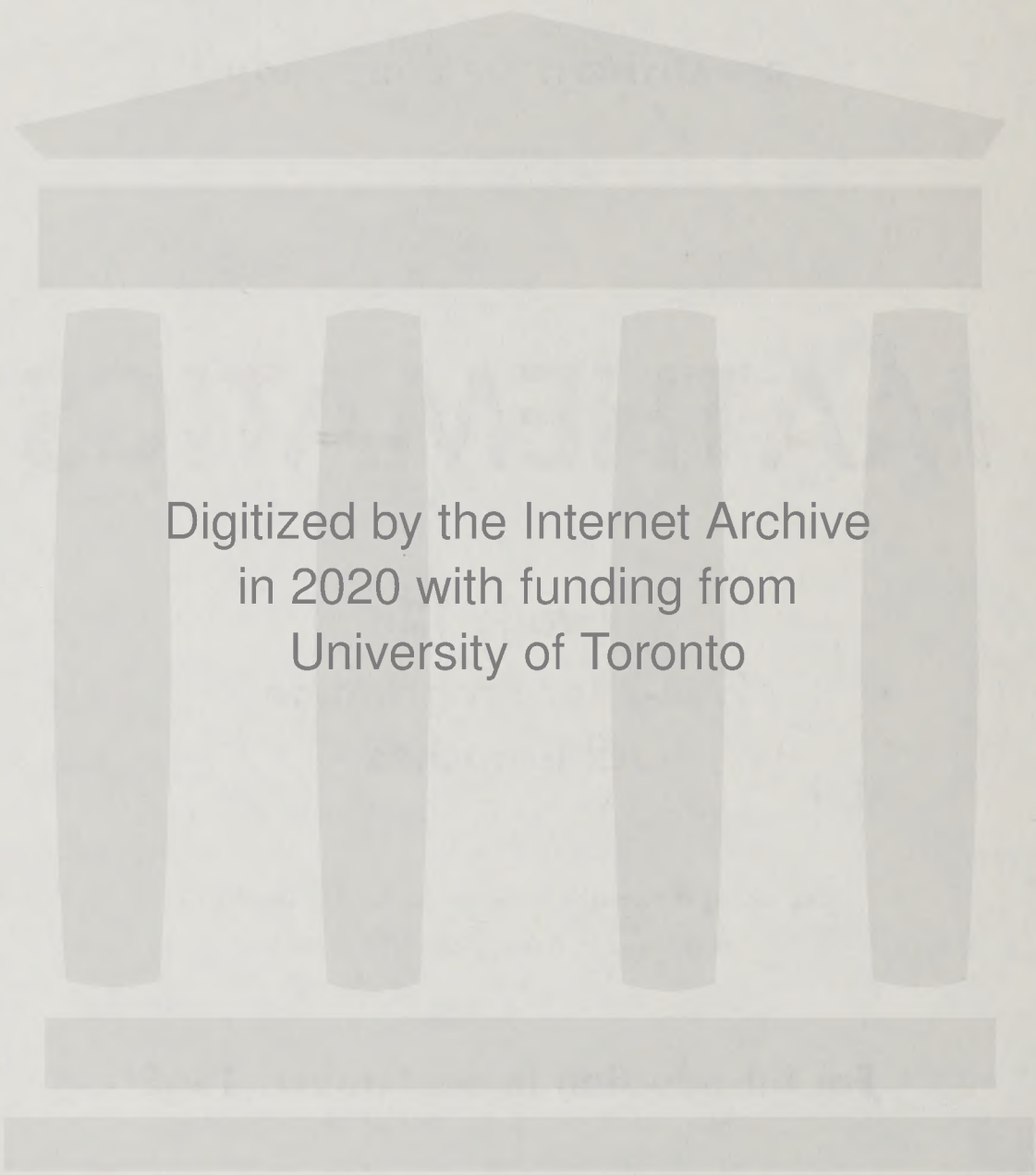
Grade 12
Five-Year Programme
All Branches

Replacing the course in Mathematics for Grade 12
as printed in Curriculum S.12 (1961)

For Introduction in September, 1965

ISSUED BY AUTHORITY OF

THE MINISTER OF EDUCATION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MATHEMATICS

Grade 12

General Observations

1. The course for Grade 12 outlined below represents the second stage in a revision of the courses for mathematics in the Senior Division of all five-year programmes in the Ontario Secondary Schools. Commencing in the school year 1965-66, it will replace the Grade 12 section (pages 15 to 18) of Curriculum S.12 "Senior Division Mathematics".

   Curriculum S.12A (Mathematics), which was released to the schools in October of 1963 for introduction in September of 1964, represents the first stage. The third stage of the revision will be an outline for Grade 13 to be issued during the school year 1964-65 for introduction in September of 1966.

2. The revised Grade 11 course is a study of the mathematics of the straight line. The Grade 12 course extends this work to the study of functions and the mathematics of the simpler curves. As in Grade 11, the graphical approach is stressed.

3. Pupils who have completed successfully the compulsory sections of the Grade 11 mathematics in Curriculum S.12A of 1963 will have the necessary prerequisites for the Grade 12 course.

4. The Grade 11 course provides some supplementary topics, particularly for those pupils who were introduced to the new approach in lower grades. These supplementary topics are of two types, those chosen from some advanced stage of the work belonging in the later basic development of the subject, and those chosen to give greater depth or to stimulate interest. Because of the nature of the problem of adjustment in Grade 11, the number of topics of the first type is large. In Grade 12, the majority of the supplementary topics are of the enrichment type.

Supplementary topics of the first type found in the Grade 11 outline must appear as compulsory topics in later grades. For this reason, those pupils who have covered these topics in Grade 11 may be repeating them in Grade 12, so it will be necessary for the teacher to take account of this fact for individuals or for classes.

5. As in Grade 11, the number of weeks to be spent on each unit is given to indicate time-planning limits and depth of treatment. It is important that attention be paid to these limits in order that proper balance and coverage of all compulsory topics may result.

   Attention is directed to the explanations accompanying the statement of certain topics. They are intended to provide guidance as to the recommended approach or level of treatment of particular topics. They are particularly important for any topics previously included in Grade 13 because these topics will usually require a different treatment in the present context.

DETAILS OF THE COURSE

UNIT 1:

RELATIONS AND FUNCTIONS (4 weeks)

This unit begins with a review of several topics from various Grade 11 units dealing with the straight line and introduces the concept of the linear function.

1. Review of the development of the real number system and its properties

   Review of graphing of linear equations and inequalities in one variable

   Solution of linear equations in one variable involving fractions

   Solution of linear equations in one variable involving literal coefficients

   Study of the number of possible solutions of $ax + b = 0$

2. Review of the rectangular Cartesian co-ordinate system, with practice in graphing ordered number pairs

   Review of the solution of linear equations in two variables, stressing the use and graphical implications

of equivalent systems of equations, and including examples with literal coefficients and a discussion of the possible number of roots

Cartesian product

3. Binary relations

The set of ordered pairs determined by a relation

The domain and range of a relation

Linear equality and inequality relations and their graphs

Determination of the intercepts

4. Function as a single-valued binary relation

Practice in the use of functional notation

Graphical study of simple functions

Examples of binary relations that are not functions

5. Examples of linear functions developed from problems in which the domain is the natural numbers, the integers, or the reals

6. The linear equation of the form $y=mx+b$

The role of each of the parameters m and b

The general linear equation $ax+by+c=0$

Review of the point-slope form, the slope y-intercept form, and the slope x-intercept form

Reduction of $ax+by+c=0$ to the slope y-intercept form

UNIT 2: THE EXPONENTIAL AND LOGARITHMIC FUNCTIONS (5 to 6 weeks)

This unit introduces a study of the graphs of the exponential and logarithmic functions, and develops the mathematical basis for the use of logarithms. Some practice in the use of logarithms for computation is included, but the stress is on the mathematical interpretations.

1. Review of the concept of power, base, exponent (or index)

   The definition of a^x where x is a positive integer and a is a real number

   The development of and practice with the laws for exponents where x is a positive integer

2. Assumption of the laws for exponents where the base is positive and the exponent is rational

   Interpretation of a^x where x is rational and a is positive, and a demonstration of the need for restricting the nature of the base

   Practice with the laws for rational exponents and positive bases

   Scientific (or standard) notation

   Practice in multiplication and division involving the use of scientific (or standard) notation

3. Assumption of the laws for exponents where the base is positive and the exponent is real

   Drawing the graphs of the exponential function defined by the equations $y=2^x$, $y=3^x$, $y=10^x$

   Study of these graphs to illustrate the rate of exponential growth

   Use of these graphs for simple computations involving multiplication, division, and roots, where the exponents must be estimated from the graph (Use of large scale graphs suggested)

   The advantage of using base 10

   Study of the exponential function using ordered pairs for $y=10^x$ from four-figure tables (antilogarithm tables)

   Use of these four-figure tables of 10^x in simple computations, including cases where $x > 1$

4. Study and comparison of the graphs of $y=10^x$ and $x=10^y$

   Study of the set of ordered pairs (x, y) satisfying $x=10^y$ where x is such that

   (i) y may be determined by inspection

   (ii) logarithm tables are necessary to determine y

Use of the notation $y=\log_{10}x$

Development of the basic properties of logarithms

Practice in computations involving the use of logarithms for products, quotients, powers, and roots using four-place tables (Computation of roots stressed)

5. Discussion of simple and compound interest

   Development of the formula for simple interest $A=P(1+ni)$

   Development of the formula for compound interest $A=P(1+i)^n$

   Practice with numerical examples involving simple and compound interest

   Study of the graphs of the functions defined by these relations

   Calculations of equivalent annual rate in cases where the compounding is semi-annual, quarterly, monthly

   Calculation of present value

6. (Supplementary)

   The mathematical theory of the slide rule as applied to multiplication, division, and square roots

UNIT 3:
QUADRATIC FUNCTIONS AND QUADRATIC EQUATIONS (6 weeks)

Throughout earlier grades, linear equations have been studied. In the first unit of Grade 12, the study of straight lines has been extended to the study of linear functions. This unit now begins a study of the characteristics and applications of quadratic functions and of the roots of quadratic equations.

1. Simple quadratic functions and their graphs to include general form, symmetry, domain, range, intercepts, type of opening, the existence of a maximum or minimum value, the regions defined by the associated inequalities

2. The quadratic equation and its graphical solution

3. Review of the factoring of quadratic polynomials
   Algebraic solutions of quadratic equations by factoring and by completing the square
   Finding maximum and minimum values of quadratic functions by completing the square

4. The general quadratic equation $ax^2+bx+c=0$ and the development of its roots
   $$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$
   Discussion of the case of non-real roots
   Recognition of the need of an extension of the number system beyond the reals

5. Word problems giving rise to quadratic equations (inadmissible solutions treated by *prior* consideration of the domain)

6. An inductive development of the factor theorem
   Equations of degree higher than the second which may be solved by factoring
   Equations involving radical signs which give rise to quadratic equations in one variable
   Equations where the variable is other than x, e.g.,
   $$6\sin^2\theta+\sin\theta-2=0$$
   $$2^{2x}-12\,(2^x)+32=0$$

7. Graphical study of the quadratic function for the cases where the roots of the corresponding equation are real and equal, real and unequal, and non-real
   The discriminant of a quadratic equation and its use in determining the character of the roots
   The sum and product of the roots of a quadratic equation and their use as a check on the roots
   The formation of a quadratic equation when given its roots

8. (Supplementary)
   (a) The general quadratic function and its graph
   (b) An inductive study of the maximum number of roots of a given equation
   (c) The graphical study of equations involving radical signs

(d) The formation of equations where the roots are the squares of, or the reciprocals of, the roots of a given equation

UNIT 4:
THE CIRCLE (6 weeks)

This unit begins with the synthetic geometry of the circle developed in the traditional manner although with some minor changes in language and organization. Then follows an establishment of analytic geometry methods for the study of the circle. Finally, the synthetic and analytic methods are linked by requiring certain problems to be solved by both methods.

1. The circle as the set of points equally distant from a fixed point

   The definitions of the following special sets of points associated with the circle: diameter, radius, chord, secant, arc (major and minor), semicircle

   The definitions of the special regions associated with the circle: interior, exterior, segment (major and minor), sector

   The measures of the circumference, arc length, and areas associated with the circle

2. Development of chord properties
   (a) The centre of a circle lies on the right bisector of any chord
   (b) The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord
   (c) Deductions dealing with
      (i) the locus of centres of circles passing through two fixed points
      (ii) circumcircles
      (iii) the relation between distances of chords from the centre and the relative lengths of the chords

3. Development of angle properties
   (a) Definition of sector angle, an angle inscribed in a circle, and an angle inscribed in a semicircle

(b) An angle inscribed in a circle is equal to one-half of the sector angle subtended by the same arc

(c) (i) angles inscribed in a circle that are subtended by the same arc are equal (and converse)

(ii) an angle inscribed in a semicircle is a right angle (and converse)

(iii) opposite angles of a quadrilateral inscribed in a circle are supplementary (and converse)

(iv) an exterior angle of a quadrilateral inscribed in a circle is equal to the interior opposite angle (and converse)

(d) Deductions applying the above properties (including as a deduction the product property of intersecting chords)

4. Development for *any* curve of the concept of tangent as the limiting position of a secant, and the relation of this concept to the circle

5. Development and applications of the tangent properties

(a) A tangent to a circle is perpendicular to the radius at the point of contact (and converse)

(b) A line is a tangent to a circle if and only if the distance from the centre of the circle to the line is equal to the radius of the circle

(c) If two tangents are drawn to a circle from an external point, the two tangents are equal, they subtend equal angles at the centre, and they make equal angles with the straight line joining the point to the centre

(d) If a chord and a tangent are drawn through a point on a circle, each of the angles determined by the tangent and the chord is equal to the angle inscribed in the circle on the opposite side of the chord

(e) The square of the length of a tangent from an external point P is equal to the product of the distances from P to the points of intersection of the circle with any secant through P

6. Development of the equation of the circle with radius r and centre (0, 0)

   A study of the relation defined by $x^2 + y^2 = r^2$ to establish domain, range, symmetry, intercepts

   Regions defined by $x^2 + y^2 < r^2$ and $x^2 + y^2 > r^2$

7. Analytic proof of particular cases involving the properties indicated in 2 (a) and 2 (b) above

8. Finding the length of a tangent from an external point to a circle whose equation is of form $x^2 + y^2 = 25$ (Particular cases — no proof of general case to be required)

9. Finding the equation of the tangent at a point on a circle whose equation is of the form $x^2 + y^2 = 49$ (Particular cases leading to a recognition of the form of the equation)

10. (Supplementary)
    Construction of circles given three conditions involving points, lines, and other circles

UNIT 5:
TRIGONOMETRIC FUNCTIONS (3 weeks)

This unit reviews and extends the trigonometry begun in Grade 11 and introduces the study of the sine and cosine functions.

1. Review of radian measure
   Review of the general definitions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ where $\theta < 2\pi$, and the determination of values in various quadrants

   Extension to values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for all real values of θ

2. The set of all ordered pairs $\left(\theta, \frac{y}{r}\right)$ and $\left(\theta, \frac{x}{r}\right)$ determined by the sine and cosine functions, respectively

   Discussion of the domain of the sine and cosine functions.

3. (a) Graphing of the sine function using a table of values corresponding to the ordered pairs $(\theta, \sin\theta)$

Consideration of periodicity and the values at or near the multiples of $\frac{\pi}{2}$

(b) Graphing of the cosine function using a table of values corresponding to the ordered pair $(\theta, \cos \theta)$

Consideration of periodicity and the values at or near the multiples of $\frac{\pi}{2}$

(c) Consideration of the period and of the amplitude of the sine and cosine functions

4. (a) Graphing of the function defined by $y=a \sin \theta$ (where a is positive) to show the effect on the amplitude when a is (i) equal to 1, (ii) less than 1, (iii) greater than 1

(b) Graphing of the function defined by $y=\sin k\theta$, where k is positive, to show that k changes the period of the curve but not the amplitude

(c) Graphing of the function defined by $y = a \sin k\theta$, where a and k are positive, to show the combined results of a change of the amplitude and of the period

(d) Graphing of the function defined by $y= a \sin (\theta + d)$, where a is positive and d is real, to show the combined results of a change of the amplitude and the phase

(e) Graphical determination of the relation between the sine and cosine functions of θ and of $\theta + \frac{\pi}{2}$.

5. Proof of the sine law and the cosine law

Problems using the sine and cosine laws restricted to cases where numbers are simple (using tables of four decimal figures for one-degree intervals and angles to nearest degree)

Use of the approximation sign

6. (Supplementary)

Application to angular velocity

Simple trigonometric identities and equations

UNIT 6:

SEQUENCES AND SERIES (3 weeks)

1. Sequence defined by means of a function where the domain is the natural numbers or a subset of the natural numbers

   Distinction between a finite and an infinite sequence

   Terms of a sequence

   General term of a sequence

   Particular sequences defined by
   (i) function notation
   (ii) the first few terms and the general term
   (iii) the general term only
   (iv) a recursion relation

2. Definition of arithmetic and geometric sequences

   The general arithmetic and the general geometric sequences

3. Definition of a series

   Inductive discovery of the sum of a finite number of terms of particular series

   The sum of n terms of the general arithmetic and the general geometric series

4. Introduction to annuities

   Amount and present value of annuities

5. (Supplementary)

   Mathematical induction